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THE ROLE OF PRESTRESSING IN ESTABLISHING REGIONS OF INSTABILITY FOR A COMPOUND COLUMN UNDER CONSERVATIVE OR NON-CONSERVATIVE LOAD

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The influence of prestressing of a non-linear two-member column on its natural vibration and stability is studied. The perturbation method is used for solving the problem. Regions of divergence and flutter instabilities for a column has been determined on the basis of courses of eigencurves in relation to prestress rate. Discontinuities of the critical force have been observed for the border value of the prestress leading to the instability of the column without the external load. Although each prestress lowers the critical force, it can be used for passive vibration control.

1. INTRODUCTION

In many engineering applications it is very important to move the natural frequencies of elastic structures far enough from a possible excitation band. This can be done by the proper selection of the physical and geometrical parameters or by introducing adequate prestress into members of the structure. Holnicki-Szulc and Haftka proposed in reference [1] the prestressing of an antenna truss to obtain reshaping of the vibration modes and obtain small amplitudes at desired points. A space structure was the object of investigation by Kwan and Pellegrino in reference [2]. They observed the role of location of actuators, their required extension as well as the best actuator adjustments in modifying an incorrect prestress rate of the structure. Przybylski *et al.* [3] demonstrated the influence of prestress, axial force as well as the distribution of both the axial and flexural rigidities on the natural frequencies of a non-linear simply supported and axially compressed two-member frame on the basis of both numerical and experimental investigations. The results obtained indicated that each prestress lowered the natural frequencies of the system which as a result can decrease the critical stability force. Generally, prestressing structures can be treated as a way of passive control of their natural vibrations. The main aim of this work is to show how prestress influences divergence and flutter instabilities

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of a simple structure composed of two identical rods loaded by conservative or non-conservative load.

The stability of non-conservative systems has been extensively studied during the last three decades. Leipholz [4] presented the state-of-the-art concerning the stability of elastic systems with classification of stability problems, indicating the necessity to use the dynamic approach for investigation of non-conservative structures. Pedersen [5] studied the cantilever follower force problem extended to a three-parameter case, including a concentrated mass, a linear elastic spring and a partial follower force at the free end. He stated that it is generally necessary to obtain the characteristic lines of instability in the load-frequency co-ordinate system to determine stability for non-conservative systems. Kounadis [6] discussed the existence of regions of divergence instability for an elastically restrained column under a follower compressive force at its end. He found a discontinuity (a jump) in the critical load, which as he stated, could be evaluated only by using the dynamic stability criterion. Bogacz and Janiszewski [7] presented a comprehensive review of the literature on the methods of analysis and optimal designing of columns subjected to follower forces. Sugiyama *et al.* [8] described the effect of an intermediate concentrated mass on the flutter instability of a cantilevered column subjected to a rocket thrust. However, internal structural damping may stabilize or destabilize a non-conservative system. It can be neglected, as done by the authors, in structures for which it is very small. Sugiyama *et al.* [8] presented experimental results that agreed well with the theoretical flutter predictions. Kurnik [9] in his book presented a thorough introduction to bifurcation in one- and two-dimensional problems and its application to divergence and flutter instability phenomenon in engineering. Kounadis [10] described the occurrence of flutter instability through Hopf bifurcation before static buckling in regions of divergence in non-conservative non-self-adjoint systems, to show that a practically non-dissipative model under certain conditions may lose its stability via flutter for a load smaller than that of divergence instability.

In this work, the dynamic approach is used to investigate the influence of prestress on a geometrically non-linear structure upon its regions of instability. The system is loaded by a subfollower force according to the follower parameter η which makes the problem a non-conservative one for $\eta > 0$ and conservative for $\eta = 0$.

2. SOLUTION OF THE PROBLEM

The scheme of deformed axes of both rods of the cantilevered structure under investigation is given in Figure 1(a). Physical models of such a structure are a column made of two coaxial tubes, or a tube and a bar (Figure 1(b)), or a planar frame made of a strip located in the centre of the structure in which the second member is formed by two identical strips, symmetrically located on both sides of the central strip (cf. reference [11])—Figure 1(c). The members of the structure after initial prestressing caused, by, e.g. the difference in their length, are rigidly connected to each other in both the displacement and rotational senses. Both

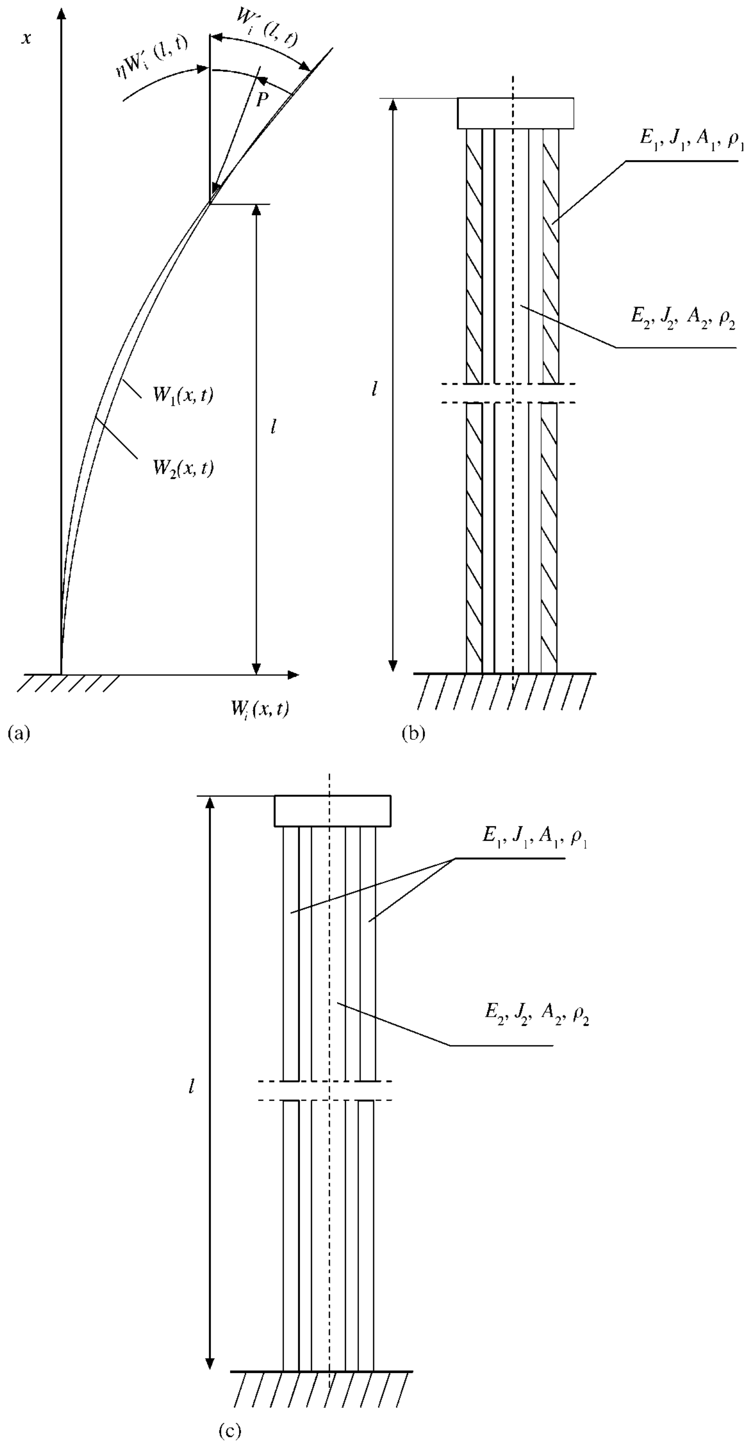

Figure 1(a). Scheme of deflected axis of rods of two-member structure under partial follower force, (b), (c). Physical models of a double member column, and a double member frame, respectively.

members in the underformed state lie in the plane perpendicular to the plane of the deflected model.

The governing equations for a double-bar frame were derived by the author and his co-workers [3] on the basis of the strain-displacement relations for a beam undergoing moderately large deflection described by von Karman and applied by Woinowsky–Krieger in reference [12], and by using Hamilton's principle presented by Levinson [13]. These equations applicable for the considered problem have got the following non-dimensional form:

— for the lateral vibration of the column ith rod:

$$\frac{\partial^4 w_i(\xi,\tau)}{\partial \xi^4} + k_i \frac{\partial^2 w_i(\xi,\tau)}{\partial \xi^2} + \varpi_{ni} \frac{\partial^2 w_i(\xi,\tau)}{\partial \tau^2} = 0 \quad (i = 1, 2), \tag{1a, b}$$

– for the longitudinal displacement $u_i(\xi,\tau)$ of the ith rod:

$$u_i(\xi,\tau) = -\frac{k_i}{\lambda_i}\xi - \frac{1}{2}\int_0^{\xi} \left[\frac{\partial w_i(\zeta,\tau)}{\partial \zeta}\right]^2 \mathrm{d}\zeta \quad (i = 1, 2), \tag{2a, b}$$

where

$$w_i(\xi,\tau) = \frac{W_i(x,t)}{l}, \quad k_i = \frac{S_i l^2}{E_i I_i}, \quad \varpi_{ni} = \Omega_n^2 l^4 \frac{\rho_i A_i}{E_i I_i} \quad (i = 1, 2) \tag{3a–c}$$

respectively denote non-dimensional transverse displacements, load parameters and non-dimensional frequency parameter, and

$$\xi = \frac{x}{l}, \quad \tau = \Omega_n t, \quad \lambda_i = \frac{A_i l^2}{I_i}, \quad u_i(\xi,\tau) = \frac{U_i(x,\tau)}{l}, \tag{3d–g}$$

where l is the length of the column, Ω_n stands for the nth natural frequency, $E_i I_i$ denotes the bending stiffness of the ith rod, and $\rho_i A_i$ stands for the mass per the unit length of the ith rod. The longitudinal force S_i has components which come from an initial prestress (which is independent of time) and from deflection occurring during vibration (which is dependent upon time). Similarly this part of the longitudinal displacement U_i which is evoked by prestress is independent of time, whereas other parts arising during vibration change with time.

By using the perturbation (small parameter) method, the relevant quantities are expanded into exponential series with respect to the amplitude parameter ε ($\varepsilon \ll 0$) (cf. Evansen [14]):

$$w_i(\xi,\tau) = \sum_{j=1}^{N} \varepsilon^{2j-1} w_{i2j-1}(\xi,\tau) + 0(\varepsilon^{N+1}), \qquad k_i = k_{i0} + \sum_{j=1}^{N} \varepsilon^{2j} k_{i2j}(\tau) + 0(\varepsilon^{N+1}),$$

$$\varpi_{ni} = \omega_{ni} + \sum_{j=1}^{N} \varepsilon^{2j} \overset{(2j)}{\omega}_{ni} + 0(\varepsilon^{N+1}), \tag{4a–c}$$

where $\overset{(2j)}{\omega}{}^2_{ni}$ stands for the frequency correction coefficients, and

$$w_{il}(\xi, \tau) = w_{il}^{(1)}(\xi)\cos\tau,$$

$$w_{i3}(\xi, \tau) = w_{i3}^{(1)}(\xi)\cos\tau + w_{i3}^{(3)}(\xi)\cos 3\tau,$$

$$w_{i5}(\xi, \tau) = w_{i5}^{(1)}(\xi)\cos\tau + w_{i5}^{(3)}(\xi)\cos 3\tau + w_{i5}^{(5)}(\xi)\cos 5\tau,$$

$$\vdots$$

$$k_{i2}(\tau) = k_{i2}^{(0)} + k_{i2}^{(2)}\cos 2\tau,$$

$$k_{i4} = k_{i4}^{(0)} + k_{i4}^{(2)}\cos 2\tau + k_{i4}^{(4)}\cos 4\tau \quad (i = 1, 2), \tag{5a–e}$$

$$\vdots$$

By introducing equations (4a–c) into the equations of motion (1a, b) and axial displacements (2a, b), and then equating the terms of the respective ε exponents to zero, one obtains the following infinite set of equations of motion and longitudinal displacements:

$$0(\varepsilon^0), \quad u_{i0}(\xi) = -\frac{k_{i0}}{\lambda_i}\xi, \tag{6a, b}$$

$$0(\varepsilon'), \quad w_{i1}^{IV}(\xi, \tau) + k_{i0} w_{il}^{II}(\xi, \tau) + \omega_{ni}\ddot{w}_{il}(\xi, \tau) = 0, \tag{7a, b}$$

$$0(\varepsilon^2), \quad u_{i2}(\xi, \tau) = -\frac{k_{i2}(\tau)}{\lambda_i}\xi - \frac{1}{2}\int_0^\xi [w_{il}^I(\zeta, \tau)]^2\, d\zeta, \tag{8a, b}$$

$$0(\varepsilon^3), \quad w_{i3}^{IV}(\xi, \tau) + k_{i0}w_{i3}^{II}(\xi, \tau) + \omega_{ni}\ddot{w}_{i3}(\xi, \tau) = -k_{i2}(\tau)w_{il}^{II}(\xi, \tau) - \overset{(2)}{\omega}_{ni}\ddot{w}_{il}(\xi, \tau) = 0$$

$$\vdots \tag{9a, b}$$

$$(i = 1, 2)$$

Roman numerals and dots denote derivatives with respect to ξ and τ respectively.

In view of equations (4a–c), equations (6–9) are to be solved under the following boundary conditions:

$$w_{1j}(0, \tau) = w_{2j}(0, \tau) = 0, \qquad w_{1j}(1, \tau) = w_{2j}(1, \tau),$$

$$w_{lj}^I(\xi, \tau)|_{\xi=0} = w_{2j}^l(\xi, \tau)|_{\xi=0} = 0, \qquad w_{1j}^l(\xi, \tau)|_{\xi=1} = w_{2j}^l(\xi, \tau)|_{\xi=1},$$

$$w_{1j}^{II}(\xi, \tau)|_{\xi=1} + \mu w_{2j}^{II}(\xi, \tau)|_{\xi=1} = 0,$$

$$w_{1j}^{III}(\xi, \tau)|_{\xi=1} + \mu w_{2j}^{III}(\xi, \tau)|_{\xi=1} + p(1-\eta)(1+\mu)w_{1j}^I(\xi, \tau)|_{\xi=1} = 0 \quad (j = 1, 3, 5, \ldots)$$

$$u_{1j}(0, \tau) = u_{2j}(0, \tau) = 0, \qquad u_{1j}(1, \tau) = u_{2j}(1, \tau)$$

$$k_{1j}(\tau)\mu_1 + k_{2j}(\tau)\mu_2 = p \quad (j = 0, 2, 4, \ldots) \tag{10a–k}$$

where

$$p = \frac{Pl^2}{E_1 I_1 + E_2 I_2}, \quad \mu_1 = \frac{E_1 I_1}{E_1 I_1 + E_2 I_2}, \quad \mu_2 = \frac{E_2 I_2}{E_1 I_1 + E_2 I_2}, \quad \mu = \frac{E_2 I_2}{E_1 I_1}, \quad \text{(11a–d)}$$

P is the external load applied to the column, and η is the follower parameter.

For the case of a column made of two identical rods as considered in this work, $E_1 I_1 = E_2 I_2 = \frac{1}{2} EI$, where EI is the bending rigidity of both rods. Similarly for the axial rigidities $E_1 A_1 = E_2 A_2 = \frac{1}{2} EA$, where EA is the axial stiffness of all system, and for masses per unit length $\rho_1 A_1 = \rho_2 A_2 = \frac{1}{2} \rho A$.

Equations (6a, b) express the axial displacement–force relation in column members. Substituting these equations into boundary conditions (10i–k) for $j = 0$, gives a linear relationship between axial forces S_{i0} in each rod due to axial prestressing with the force P_1 and the external force P in the following form;

$$S_{i0} = \pm P_1 + \tfrac{1}{2} P \quad (i = 1, 2). \quad \text{(12)}$$

The force P_1 is taken as positive when it compresses a particular rod. For an externally unloaded column ($P = 0$) when one member is compressed the second must be stretched by the same P_1. Taking into account different ways of structure loading, the distribution of internal forces can differ, which strongly influences both the vibration frequency and stability of the system.

The general solution of equations (7a, b), after separation of the ξ and τ variables according to equation (5a) is as follows:

$$\overset{(1)}{w}_{i1}(\xi) = A_{i1} \cosh(\alpha_{i1} x) + B_{i1} \sinh(\alpha_{i1} x) + C_{i1} \cos(\beta_{i1} x) + D_{i1} \sin(\beta_{i1} x) \quad (i = 1, 2), \quad \text{(13a, b)}$$

where

$$\alpha_{i1} = \sqrt{-\tfrac{1}{2} k_{i0} + \sqrt{\tfrac{1}{4} k_{i0}^2 + \omega_{ni}}}, \quad \text{(14a, b)}$$

$$\beta_{i1} = \sqrt{\tfrac{1}{2} k_{i0} + \sqrt{\tfrac{1}{4} k_{i0}^2 + \omega_{ni}}}. \quad \text{(14c, d)}$$

By substituting equations (13a, b) into boundary conditions (10a–h) for $j = 1$ one obtains a system of eight homogeneous equations with unknown integration constants A_{i1}, B_{i1}, C_{i1} and D_{i1} $(i = 1, 2)$. The determinant of the matrix of coefficients of the system must be equal to zero to get a non-trivial solution of the problem. This expresses the relationship between the load and the natural frequency and is solved numerically. To find the vibration mode for the calculated frequency a normalization condition was applied setting $\overset{(1)}{w}_{i1}(1) = 1$. The equations resulting from this condition substitutes an arbitrary one in the system of eight homogeneous equations described above. Numerical solution of the new system of equations gave the values of constants A_{i1}, B_{i1}, C_{i1} and D_{i1} $(i = 1, 2)$ from equations (13a, b) expressing mode shapes of both rods.

3. AMPLITUDE–FREQUENCY RELATION FOR THE CONSERVATIVE LOAD (CASE $\eta = 0$)

The amplitude force parameters $k_{i2}(\tau)$ existing in equations (8a, b) depends on the vibration amplitude. They were derived from conditions (10j) by taking into consideration equations (5a) and (5d) as well as condition (10k). This leads to the equations

$$k_{12}^{(0)} = k_{12}^{(2)} = \frac{1}{4}\frac{\mu\lambda_1\lambda_2}{\lambda_1+\lambda_2}\int_0^1\left(\left[\frac{\partial_{w21}^{(1)}(\xi)}{\partial\xi}\right]^2 - \left[\frac{\partial_{w11}^{(1)}(\xi)}{\partial\xi}\right]^2\right)\mathrm{d}\xi, \tag{15a}$$

$$k_{22}^{(0)} = k_{22}^{(2)} = -\frac{1}{\mu}k_{12}^{(0)}. \tag{15b}$$

It is easy to find from the above equations that amplitude force parameters $k_{12}^{(0)}$ and $k_{22}^{(0)}$ differ from zero if the vibration modes of both rods do not overlap each other, or if there appear no antisymmetric modes characterized by the same amplitude for each rod and the opposite sign of the curvature. For our case of a column made of identical rods, non-zero force parameters $k_{12}^{(0)}$ and $k_{22}^{(0)}$ exist only for the prestressed column. These parameters were calculated numerically after analytical solution was performed, i.e., when equations (13a, b) were inserted into equation (15a). The internal force parameter in each rod is now equal to

$$k_i(\tau) = k_{i0} + \varepsilon^2 \overset{(0)}{k}_{i2}(1+\cos 2\tau) \tag{16}$$

and is dependent on the value of the amplitude parameter ε.

The frequency correction parameter $\omega_{ni}^{(2)}$ from equations (9a, b) can be found from the orthogonality condition proposed by Keller and Ting [15]. For the conservative load of column ($\eta = 0$), equation (9) (for each $i = 1, 2$) is the inhomogeneous form of adequate equation (7) and both equations have similar boundary conditions. Equation (9) will have periodic solution if and only if its right-hand side is orthogonal to all solutions of the adjoint homogeneous equation (7). After the separation of ξ and τ according to equations (5b) and (5d), the orthogonality condition is obtained by multiplying equations (9a, b) by $E_iJ_iw_{i1}^{(1)}$, integrating them over the range $\langle 0, 1\rangle$ and adding with respect to i ($i = 1, 2$). It is easy to find that the integral of the left-hand side vanishes when conditions (10a–h) for $j = 1, 3$ are taken into account. The right-hand side of equations (9a, b) yields

$$\frac{3}{2}k_{12}^{(0)}\int_0^1\left(\left[\frac{\partial\overset{(1)}{w}_{11}(\xi)}{\partial\xi}\right]^2 - \left[\frac{\partial\overset{(1)}{w}_{21}(\xi)}{\partial\xi}\right]^2\right)\mathrm{d}\xi + \overset{(2)}{\omega}_{ni}\int_0^1\left([\overset{(1)}{w}_{11}(\xi)]^2 + [\overset{(1)}{w}_{21}(\xi)]^2\right)\mathrm{d}\xi = 0. \tag{17}$$

Inserting (15a) into (17) one obtains

$$\omega_{ni}^{(2)} = \frac{3\lambda_1}{16}\frac{\left(\int_0^1([\partial\overset{(1)}{w}_{21}(\xi)/\partial\xi]^2 - [\partial\overset{(1)}{w}_{11}(\xi)/\partial\xi]^2)\,\mathrm{d}\xi\right)^2}{\left(\int_0^1[\overset{(1)}{w}_{11}(\xi)]^2 + [\overset{(1)}{w}_{21}(\xi)]^2\right)\mathrm{d}\xi}. \tag{18}$$

The first correction frequency parameter $\overset{(2)}{\omega}_{ni}$ takes non-zero values when both rods of column vibrate with different mode shapes. Having this parameter numerically calculated after analytical solution of equation (18) the frequency according to (4e–f) is

$$\varpi_{ni} = \omega_{ni} + \varepsilon^2 \overset{(2)}{\omega}_{ni} + 0(\varepsilon^4). \tag{19}$$

The effects of non-linearity in the problem begin with the terms $0(\varepsilon^2)$ and so the changes in vibration amplitude controlled by ε (cf. equation (4a, b)) affect the amplitude force parameter $\overset{(0)}{k}_{12}$ and the frequency correction parameter $\overset{(2)}{\omega}_{ni}$. Amplitude–frequency relation which exists for non-linear problems can be exhibited for arbitrary level of load if at least the first frequency correction parameter is calculated. It is customary in practice to restrict to terms up to an order of two in expansion (4e, f) for problems of vibration in lower modes (cf. references [14, 16]). The higher order terms need be considered for the response of higher modes.

For non-conservative load ($\eta > 0$) the boundary-value problem is non-self-adjoint and so the construction of the adjoint system is necessary to obtain amplitude–frequency relation (cf. are reference [17]). For the case of the Beck problem, which can be treated as an unprestressed two-rod column subjected at its free end to a compressive follower force, the Reut problem is the adjoint one. Because of the volume of present work, the problem of amplitude–frequency relation for a non-conservatively loaded two-bar column will be shown in a subsequent paper.

4. RESULTS OF NUMERICAL CALCULATION

All results are presented as functions of dimensionless quantities in a way to compare them with results obtainable for a single-rod column of the bending rigidity equal to EI and masses per unit length equal to ρA. These quantities are as follows

— $p = Pl^2/EI$: the external load parameter; p_c is the critical value of p, $p_c^{(B)}$ is the critical parameter for a Beck's column equal to 20·0509;
— $p_{ml} = P_1 l^2/EI$: the internal prestress parameter (when positive, the rod 1 is compressed and the rod 2 is stretched),
— $\omega_n = \rho A \Omega_n^2 l^4/EI$: the natural vibration frequency parameter.

Figure 2 shows the natural frequency curves obtained for zero prestress and different follower parameter. These curves are identical to those for an analogically supported and loaded single-rod column of bending rigidity EI and mass per unit length ρA. For $\eta = 1$ the flutter critical load appears for exactly the same value as for Beck's column [18], ($p_c^{(B)} = 20{\cdot}0509$), because both the identical rods due to equality of their rigidities and masses per unit length vibrate with the same first and second frequencies and mode shapes as a single column. For $\eta = 0{\cdot}5$ the curve is tangent to the p-axis at the point of column divergence instability ($p_c = 9{\cdot}9002$). For $\eta = 1{\cdot}5$, the case of anti-tangential load, the critical flutter load is higher than for

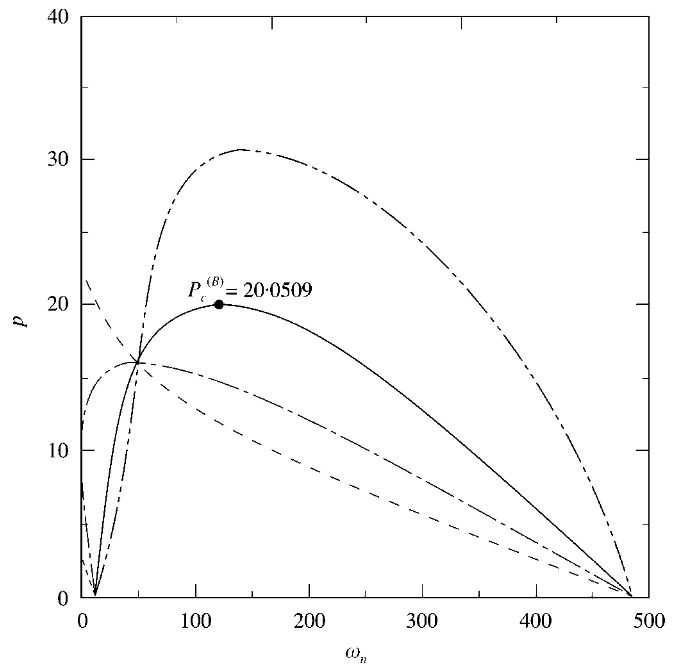

Figure 2. Natural vibration curves for an unprestressed column and different follower parameter η. $p_{m1} = 0$; - - - - -, $\eta = 0$; —-—--, $\eta = 0{\cdot}5$; ——, $\eta = 1{\cdot}0$; —--, $\eta = 1{\cdot}5$.

$\eta = 1$. When $\eta = 0$ the system is conservative and the column loses its stability via divergence at points where frequency curves cross the p-axis; the first critical load p_c is equal to 2·4674 and the second one also shown in Figure 2 is equal to 22·2066. For the external load $p = 0$ all curves start from the same points on the frequency axis.

The influence of internal prestressing on the natural frequency curves for conservatively loaded column ($\eta = 0$) is shown in Figure 3. All basic computation were performed for $\varepsilon = 1 \times 10^{-3}$. This value of the amplitude parameter was chosen on the basis of author's experience in amplitude measurements during experimental investigation of plane frames and columns presented in references [19, 20]. During calculation the prestress rate was a linear function of the dimensionless Beck's load $p_c^{(B)} = 20{\cdot}0509$. For this type of load ($\eta = 0$) the column exhibits only divergence type instability, which leads to a conclusion that prestress cannot change the type of instability of the conservative system and affects only its critical force. The p-axis in Fig. 3 has the logarithmic scale to show how values of the critical load decrease to zero with the growth in the prestress rate up to $0.5579122p_c^{(B)}$. Then a jump phenomenon in the value of the critical load occurs as shown in Figure 4. This is caused by the fact that the first natural frequency curve begins very close to the origin and goes to the second quadrant beyond the p-axis in Figure 3 (frequencies ω_n take negative values), whereas the second frequency curve intersects this axis at the point of the critical force of higher value. Further increase in prestress causes decrease in the critical load. The amplitude–frequency relation for differing

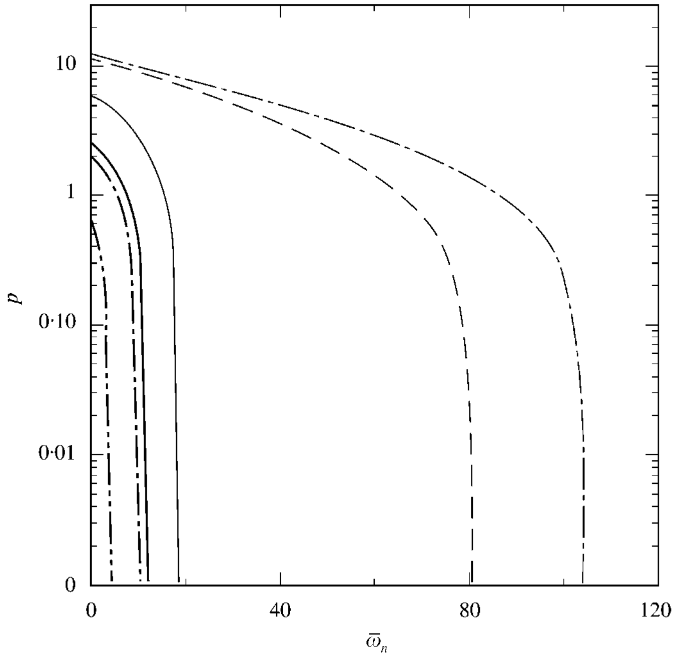

Figure 3. Natural vibration curves for a column with different prestress rates and loaded by a conservative force ($\eta = 0$, $\varepsilon = 0{\cdot}001$). $\eta = 0$; $\varepsilon = 0{\cdot}001$. ——, $p_{m1} = 0$; — - ——-, $p_{m1} = 0{\cdot}3\ p_c^{(B)}$; —--, $p_{m1} = 0{\cdot}5\ p_c^{(B)}$; ———--, $p_{m1} = 0{\cdot}56\ p_c^{(B)}$; ———-, $p_{m1} = 0{\cdot}6\ p_c^{(B)}$; ——, $p_{m1} = 1{\cdot}0\ p_c^{(B)}$.

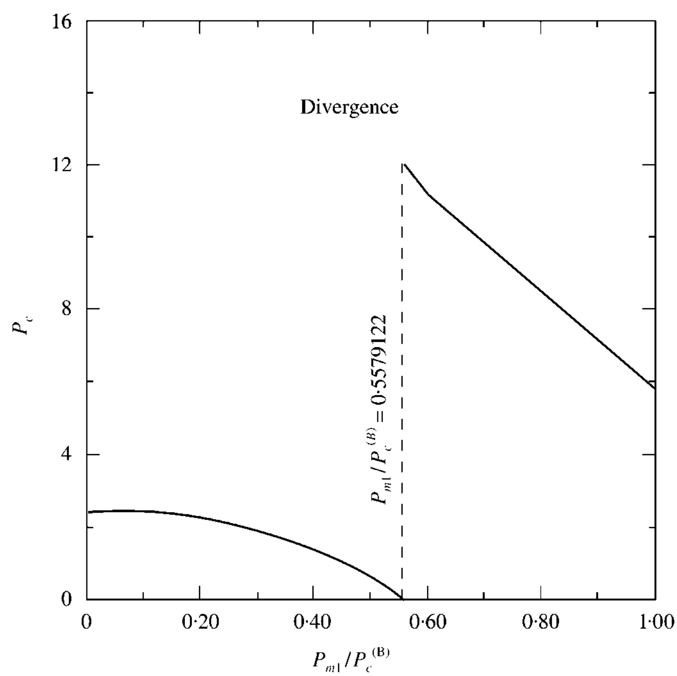

Figure 4. Regions of divergence instability for different prestress rates for a column under conservative load ($\eta = 0$, $\varepsilon = 0{\cdot}01$).

TABLE 1

Amplitude–frequency relation for a prestressed column under conservative load

$\frac{\varepsilon w_{i1}(1)}{r/l}$	$\frac{\varpi_{nl}}{\omega_{nl}}$, $\frac{p_{ml}}{p_c^{(B)}} = 0{\cdot}25$, $\frac{p}{p_c} = 0{\cdot}2$	$\frac{p_{ml}}{p_c^{(B)}} = 0{\cdot}25$, $\frac{p}{p_c} = 0{\cdot}4$	$\frac{p_{ml}}{p_c^{(B)}} = 0{\cdot}25$, $\frac{p}{p_c} = 0{\cdot}6$	$\frac{p_{ml}}{p_c^{(B)}} = 0{\cdot}75$, $\frac{p}{p_c} = 0{\cdot}2$	$\frac{p_{ml}}{p_c^{(B)}} = 0{\cdot}75$, $\frac{p}{p_c} = 0{\cdot}4$	$\frac{p_{ml}}{p_c^{(B)}} = 0{\cdot}75$, $\frac{p}{p_c} = 0{\cdot}6$
1	1·0001	1·0002	1·0047	1·0116	1·0085	1·0085
2	1·0005	1·0091	1·0013	1·0465	1·0341	1·0341
3	1·0014	1·0021	1·0039	1·1046	1·0769	1·0767
4	1·0023	1·0037	1·0069	1·1860	1·1367	1·1364
5	1·0036	1·0059	1·0108	1·2906	1·2136	1·2131

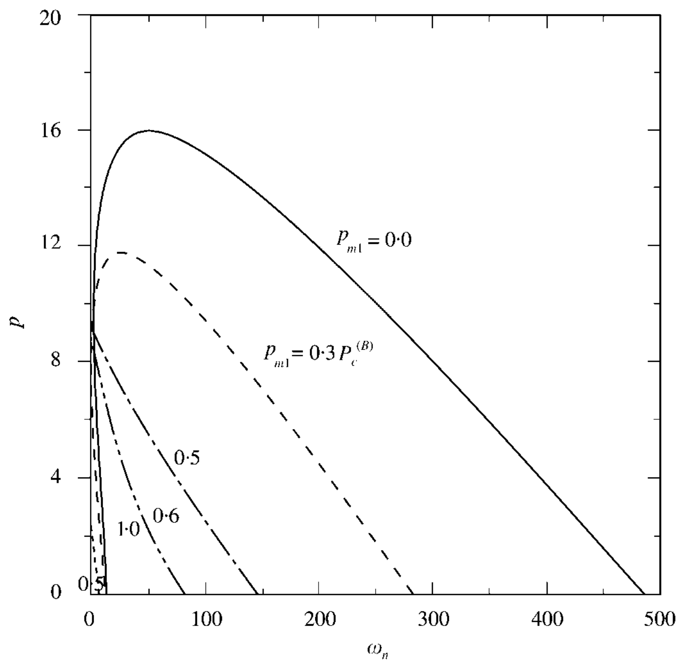

Figure 5. Natural vibrations curves for a column with different prestress rates and loaded by a non-conservative force ($\eta = 0{\cdot}5$). ——, $p_{m1} = 0{\cdot}0$; - - - -, $p_{m1} = 0{\cdot}3\ p_c^{(B)}$; —-—-, $p_{m1} = 0{\cdot}5\ p_c^{(B)}$; ———-, $p_{m1} = 0{\cdot}6\ p_c^{(B)}$; ——-—, $p_{m1} = 1{\cdot}0\ p_c^{(B)}$.

prestress rates p_{m1} and external load p is shown in Table 1. The amplitude was presented in relation to the non-dimensional radius of gyration ($r = \sqrt{J/A}$) after Evansen [14]. When the column is slightly prestressed the growth in the frequency is small, the effect of non-linearity being more visible for greater prestress of the system.

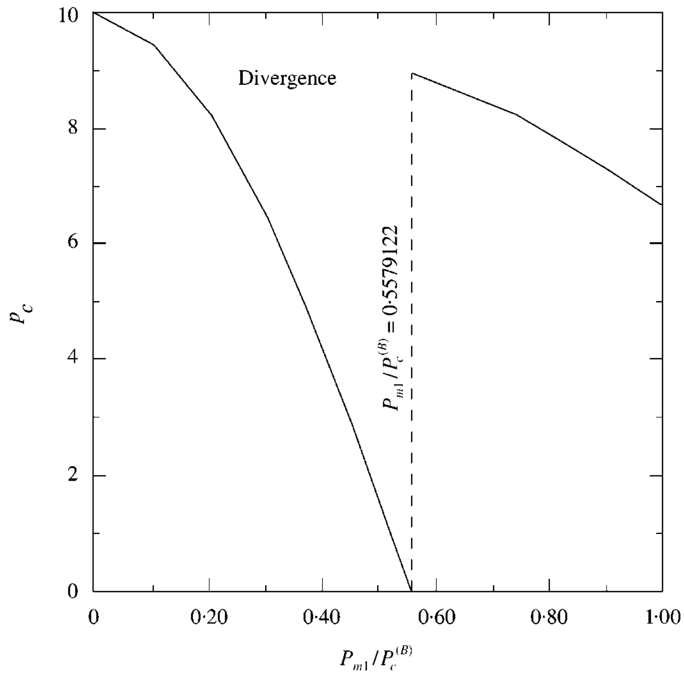

Figure 6. Regions of divergence instability for different prestress rates for a column under non-conservative load ($\eta = 0{\cdot}5$).

Special attention should be focused on the border value of prestress $p_{m1} = 0{\cdot}5579122 p_c^{(B)}$, for which for zero external load the frequency ω_{11} is numerically equal to zero ($\omega_{11} \approx 5 \times 10^{-6}$). The system for zero external load is conservative and so the orthogonality condition has the same form for any η. As a result, all frequency curves presented in the subsequent figures begin at this same points on the frequency axis. For $p = 0$ the first frequency correction parameter $\overset{(2)}{\omega}_{1i}$ is not exactly zero, because for $\omega_{11} \approx 5 \times 10^{-6}$ mode shapes of both rods differ from each other and can be calculated depending on the amplitude parameter ε. According to (4a, b) and (4e, f) mode shapes depend linearly on ε, whereas frequency increases with a square of ε. Therefore, for $\varepsilon w_{i1}(1) = 1 \times 10^{-3}$, the frequency $\varpi_{1i} = \varepsilon^2 \overset{(2)}{\omega}_{1i} = 8{\cdot}9 \times 10^{-4}$, but for $\varepsilon w_{1i}(1) = 1 \times 10^{-2}$, the frequency $\varpi_{1i} = \varepsilon^2 \overset{(2)}{\omega}_{1i} = 8{\cdot}9 \times 10^{-2}$. As a consequence, the border value of prestress can be slightly higher for the greater amplitude considered.

The effect of prestress on the natural vibration curves for the column under subfollower force ($\eta = 0{\cdot}5$) is shown in Figure 5. For this case the column loses its stability only via divergence. Prestress up to $p_{m1} = 0{\cdot}5579122 p_c^{(B)}$ diminishes the value of the critical force; then a jump in the divergence load appears and further decrease in this force is associated with increase of the prestress (Figure 6.)

Another pattern of changes in the natural frequency curves exists for the prestressed column loaded by the follower force ($\eta = 1{\cdot}0$) as shown in Figure 7. For $p_{ml} = 0$ up to $p_{m1} = 0{\cdot}5579122 p_c^{(B)}$ each pair of two first natural frequency curves

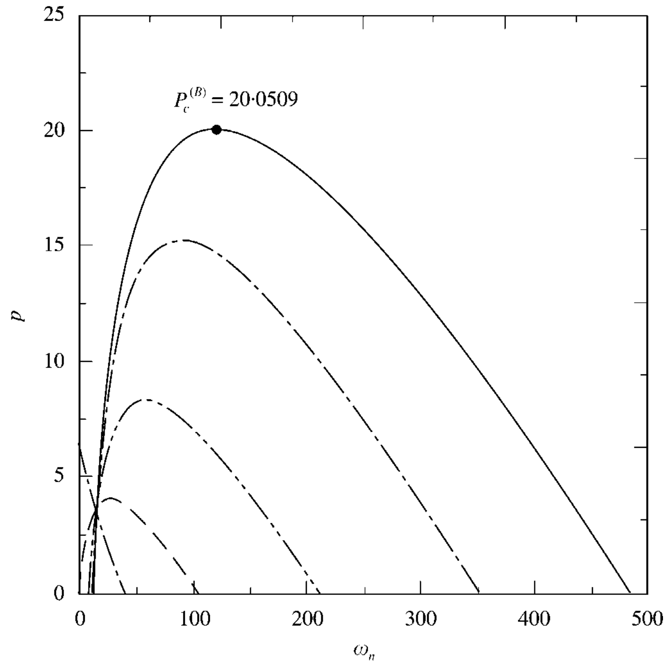

Figure 7. Natural vibration curves for a column with different prestress rates and loaded by a non-conservative force ($\eta = 1{\cdot}0$). ——, $p_{m1} = 0$; – – – –, $p_{m1} = 0{\cdot}2\ p_c^{(B)}$; —- - -, $p_{m1} = 0{\cdot}4\ p_c^{(B)}$; — —, $p_{m1} = 0{\cdot}5579122\ p_c^{(B)}$; — - —·, $p_{m1} = 0{\cdot}7\ p_c^{(B)}$.

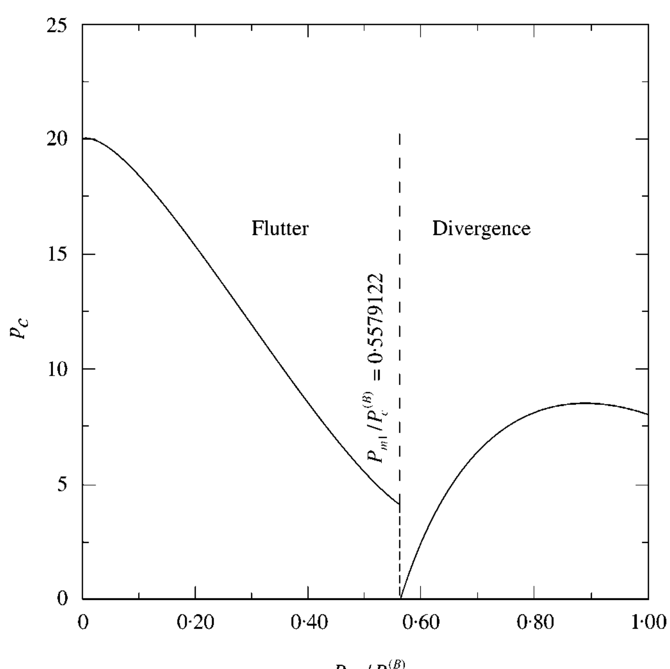

Figure 8. Regions of flutter and divergence instabilities for different prestress rates for a column under non-conservative load ($\eta = 1{\cdot}0$).

coincides at the point of flutter instability; however the first frequency curve for $p_{m1} = 0{\cdot}5579122 p_c^{(B)}$ begins at the origin which means that at this point divergence instability exists. Further increase in prestress gives the growth in the divergence critical force. The regions of instability for $\eta = 1{\cdot}0$ are presented in Figure 8. In this case of load not only the values of the critical force but also the way of instability from flutter to divergence change.

4. CONCLUSIONS

On the basis of dynamic analysis, regions of instability as a function of an initial prestress for a non-linear two-rod column has been established. An unprestressed column constructed of two identical rods vibrates like a single column of adequate physical properties.

An evolution in the course of the frequency curves of a prestressed column can be observed for increasing values of the follower parameter from the range of $\eta \in \langle 0, 1 \rangle$. At an applied load of zero the system is a conservative one and all consecutive frequency curves begin at the same points independently of the value of η.

The addition of prestress may destabilize a conservatively or non-conservatively loaded system in certain ranges. There is a border value of prestress for all the investigated range of the follower parameter, for which a jump phenomenon appears diminishing the critical load to zero.

A column without prestress loaded by a follower force loses its stability via flutter. Introduction of prestress may change not only the critical load, but also the instability mechanism from flutter to divergence.

Introduction of the adequate prestress into rods of column can be applied for passive vibration control.

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